BACKPAPER EXAMINATION - COMMUTATIVE ALGEBRA - MMATH - 30 DECEMBER 2010

Attempt all questions. All rings considered are commutative with 1. All questions carry equal marks. Total Marks - 50. Time - 3 hrs.

(1) Let A be a ring and P be a prime ideal of A. Let A_P be the localization $S^{-1}A$ where $S = A - P$, and let PA_P be the unique maximal ideal of A_P. Prove that A_P/PA_P is isomorphic to the quotient field of A/P.
(2) Let A be a Noetherian ring and P be a prime ideal of A. Prove that A_P is an Artinian ring, if and only if, P is a minimal prime ideal.
(3) Let A be the ring of rational functions of z with complex coefficients having no pole on the circle $|z| = 1$. Is A a Noetherian ring? Justify your answer.
(4) Let $f : B \to B'$ be a homomorphism of A-algebras, and let C be an A-algebra. If f is integral, prove that $f \otimes 1 : B \otimes_A C \to B' \otimes_A C$ is integral.
(5) If I is a radical ideal (that is, $I = \sqrt{I}$), then I has no embedded primes ideals.